PROPAGATING MODES ALONG A THIN WIRE LOCATED NEAR A GROUNDED DIELECTRIC SLAB

Edward F. Kuester and David C. Chang
Dept. of Electrical Engineering
University of Colorado
Boulder, Colorado, 80309

Abstract

The mode supported by a wire located parallel to a grounded dielectric slab are investigated. While at low frequencies, a "quasi-TEM" behavior is exhibited, it is shown numerically that under certain conditions, a very different "surface-attached" character appears. These results indicate the possibility of similar behavior occurring in the related configuration of open microstrip.

Introduction

It has recently been demonstrated [1,2] that a thin horizontal wire located parallel to a conducting earth can support a so-called "earth-attached" mode in addition to the well-known "transmission-line" mode which becomes TEM in the limit of a perfectly conducting earth. The apparent physical mechanism which gives rise to this new mode seems to be an interaction of the wire with the Zenneck surface wave of the air-earth interface. If, instead of a semi-infinite earth, the wire is located near a grounded dielectric slab (Fig.1), it seems possible that a similar interaction could occur with the TM mode of the slab (whose surface wave character is more pronounced than that of the Zenneck wave), and that a second mode could be generated in this case as well. Because of its structural similarity to that of microstrip, while being simpler to formulate mathematically, it was decided to investigate this configuration to decide whether such a phenomenon could be expected in the latter case before actually performing the analysis. The only previous related work seems to have been an investigation of a wire centered in an ungrounded slab [3], but in this case no TEM mode exists in the low-frequency limit.

Formulation of the Modal Equation

Let the thickness of the slab be t, and its relative permittivity $\varepsilon_r = n^2$. The wire, whose radius is a, is located at a height d above the surface of the slab. The modal equation can be derived in a thin wire approximation by following a procedure similar to that of Wait [4] for the wire over earth. If propagation of the form $\exp(ik_o\alpha z - i\omega t)$ is assumed, where $k_o = \omega(\mu_o\varepsilon_o)^{\frac{1}{2}}$, the z-axis coincides with the axis of the wire, and α is a normalized propagation constant, we obtain

$$\zeta^2[H_o^{(1)}(\zeta A) - H_o^{(1)}(2\zeta D)] + F(\alpha) = 0 \qquad (1)$$

where

$$A = k_o a; \quad D = k_o d; \quad \zeta = (1-\alpha^2), \ \text{Im}\,\zeta \geq 0$$

$H^{(1)}$ is the Hankel function of first kind, and

$$F(\alpha) = \frac{2}{i\pi}\int_{-\infty}^{\infty} \frac{[\zeta_n^2 u_1 \cosh u_2T + \zeta^2 u_2 T]\sinh u_2 T \; e^{-2u_1D}}{[u_1 \sinh u_2T + u_2\cosh u_2T][n^2 u_1\cosh u_2T + u_2 \sinh u_2T]} d\lambda \qquad (2)$$

where

$$T = k_o t; \quad \zeta_n = (n^2-\alpha^2)^{\frac{1}{2}}, \quad \text{Im}\,\zeta_n \geq 0$$

$$u_1 = (\lambda^2-\zeta^2)^{\frac{1}{2}}, \quad \text{Re}\,u_1 \geq 0; \quad u_2 = (\lambda^2-\zeta_n^2)^{\frac{1}{2}}, \ \text{Re}\,u_2 \geq 0$$

As $n \to \infty$ or $T \to 0$, it can be shown that $F(\alpha) \to 0$, and thus the first two terms of (1) represent the effect of the wire and a perfect image at a distance d below the surface of the slab. The solution to (1) in either of these limits is $\zeta = 0$ or $\alpha = \pm 1$, and the mode is the TEM mode on the wire-image transmission line.

$F(\alpha)$ represents the effect of the slab, and in fact the first term in brackets in the denomination of (2) is, when set equal to zero, the modal equation for TE surface waves of the slab, while the second brackets is that for TM surface waves.

If the condition $(n^2-1)^{\frac{1}{2}}T \ll 1$ is satisfied, there are no TE modes on the slab, and only one TM modes (thus one pair of poles in the integrand of (2)). Under the additional constraint $(n^2-1)^{\frac{1}{2}}D \geq 1$, the damping influence of the exponential in (2) allows the hyperbolic functions to be approximated by their small argument forms, giving

$$F(\alpha) \simeq \frac{2}{i\pi}\int_{-\infty}^{\infty} \frac{\zeta_n^2 u_1 + \zeta^2 u_2^2 T}{(u_1+1/T)(n^2u_1+u_2^2T)} e^{-2u_1D} d\lambda \qquad (3)$$

which, under the conditions above further reduces to

$$F(\alpha) \simeq \frac{2}{i\pi}\int_{-\infty}^{\infty} \left[\frac{1}{u_1+1/T} - \frac{\alpha^2}{u_1+n^2/T} + \frac{\alpha^2\beta^2}{u_1-\beta}\right] e^{-2u_1D} d\lambda \qquad (4)$$

where

$$\beta = (n^2-1)T/n^2 \qquad (5)$$

is the approximate location of the TM surface wave value for u_1. Further approximations valid under the same conditions give

$$\frac{2}{i\pi}\int_{-\infty}^{\infty} \frac{e^{-2u_1D}}{u_1+1/T} d\lambda \simeq \frac{2}{i\pi}\int_{-\infty}^{\infty} \frac{u_1T}{u_1(1+u_1T)} e^{-2u_1D} d\lambda$$
$$\simeq \frac{2}{i\pi}\int_{-\infty}^{\infty} \frac{\sinh u_1T}{u_1} e^{-u_1(2D+T)} d\lambda = H_o^{(1)}(2\zeta D) - H_o^{(1)}(2\zeta(D+T)) \qquad (6)$$

and likewise

$$\frac{2}{i\pi}\int_{-\infty}^{\infty} \frac{e^{-2u_1D}}{u_1+n^2/T} d\lambda \simeq H_o^{(1)}(2\zeta D) - H_o^{(1)}(2\zeta(D+T/n^2)) \qquad (7)$$

Integrals similar to the last term in (4) are encountered in the wire over earth problem [2,5,6], where methods for their approximate evaluation are given. For the particular case when $|\zeta D| \ll 1$, we obtain by these methods

$$\frac{2}{i\pi}\int_{-\infty}^{\infty} \frac{e^{-2u_1D}}{u_1-\beta} d\lambda \simeq 2H_o^{(1)}(2\zeta D) + \frac{4}{i\pi}\frac{\beta}{\sqrt{\alpha^2-\alpha_p^2}}\left[\pi - \tan^{-1}\frac{\sqrt{\alpha^2-\alpha_p^2}}{\beta}\right] \qquad (8)$$

where

$$\alpha_p = (1+\beta^2)^{\frac{1}{2}} \qquad (9)$$

denotes the location of the lowest order TM mode of the slab; just as modes with $\alpha^2<1$ must be leaky since they radiate into the free space above the slab, modes with $1 < \alpha^2 < \alpha_p^2$ must radiate into the TM surface wave away from the wire, and are thus also leaky. The reader's attention is called to the singularity of (8) at $\alpha = \alpha_p$, which can in principle be a large contribution to the modal equation.

Combining (1), (4), (6), (7) and (8), we obtain the approximate modal equation, after the small argument forms of $H_o^{(1)}$ are taken:

$$0 = \alpha^2 \ln \frac{2(D+T/n^2)}{A} - \ln \frac{2(D+T)}{A} + 2\alpha^2\beta^2 \left\{ \ln D\sqrt{\alpha^2-1} + \gamma - \frac{\beta}{\sqrt{\alpha^2-\alpha_p^2}}\left[\pi - \tan^{-1}\frac{\sqrt{\alpha^2-\alpha_p^2}}{\beta}\right] \right\} \tag{10}$$

where $\gamma = 0.5772...$ is Euler's constant. For two special cases, perturbation solutions of (10) can be found:

1) α not close to α_p: Here the term in curly brackets in (10) can be neglected because of the presence of the small factor β^2. The perturbation solution to (10) is thus simply

$$\alpha^2 \simeq \frac{\ln[2(D+T)/A]}{\ln[2(D+T/n^2)/A]} \simeq 1 + \frac{\beta}{D\ n(2D/A)} \tag{11}$$

This is the quasi-TEM approximation; in order for it to be a proper mode ($\alpha^2 > \alpha_p^2$) we must have

$$\beta D \ln(2D/A) < 1 \tag{11a}$$

which is certainly true for sufficiently low frequencies.

2) α close to α_p: Let us assume that $\sqrt{\alpha^2-\alpha_p^2} = \beta^2\delta$, where $\delta = O(1)$ is a positive number. Now the singular part of (10) dominates, and we obtain a "surface-attached" approximation

$$\delta \simeq \frac{2\pi}{\beta[\ln(2D/A) + 2\ln\beta D + 2\gamma + 2] - 1/D} \tag{12}$$

and since $\delta > 0$ we require

$$\beta D[\ln(2D/A) + 2\ln\beta D + 2\gamma + 2] > 1 \tag{12a}$$

Under the present assumptions, it seems unlikely that (11a) and (12a) could be satisfied simultaneously (so that only one mode exists), but if A is sufficiently small, (12a) can be satisfied and a region is entered in which the quasi-TEM theory is insufficient.

Numerical Results and Discussion

In order to test the validity of the approximate modal equation (10), its numerical solution was compared with a numerical solution of the exact modal equation (1) using $F(\alpha)$ as given by (2). The latter (E) as well as the solution of (10), (A), are shown in Fig. 2 for $n = 1.5$ and $D = 1.0$. Reasonable agreement is found over the entire range of T, but, as is to be expected from the nature of the approximations, the best agreement is for $T < 0.1$. The prediction of the quasi-TEM theory (11) is also shown; for $T > 0.1$, the error in $(\alpha-1)$ is measured in hundreds of per cent. The reason for this failure of quasi-TEM theory can be seen by inspecting (10). For small enough T, condition (11a) holds and α_p is sufficiently far from α to leave (11) unaffected. However, as can be seen in Fig. 2, α as given by (11) soon becomes less than α_p, while the actual value of α is "dragged" upwards by the influence of α_p. Although the value of α given by (12) turns out to be a rather crude estimate for the cases considered here, it gives the qualitatively correct behavior, and clearly for values of T larger than about 0.2, the mode is heavily influenced by the TM surface wave of the slab, and is no longer even approximately a TEM-like mode. It is thus to be expected that the fields of the mode, when it has attained this "surface-attached" character, will spread out along the slab away from the wire to a much greater extent than those of a quasi-TEM mode. Similar behavior has been found in the wire over earth problem [1,2].

Figure 3 shows results for a substrate of higher refractive index ($n = 3$); similar behavior to that of Figure 2 is found. In Figure 4, the (rather small) effect of the value of D on the solution of (10) is shown.

Conclusion

The numerical results presented here gave no evidence of the existence of a second mode, as was found for the wire over earth problem, but do indicate that strong interaction with the TM surface wave of the slab can occur as the electrical thickness of the slab becomes significant. The singular term in (10) which reflects this coupling can thus not be neglected in this parameter range. A similar singular term has been found in an analysis of microstrip [7] but was neglected in solving the modal equation; other recent treatments of stripline have also retained its influence implicitly [8,9]. The explicit display of this term gives physical insight into the reason why neither TEM nor quasi-TEM theory is sufficient for such waveguides operating in this parameter range.

Acknowledgments

This research was supported by the Office of Naval Research under contract no. N0014-76-C-0318.

References

1. R.G. Olsen and D.C. Chang, Electron. Lett. v.10, pp. 92-94 (1974).

2. R.G. Olsen and D.C. Chang, Sci. Rept. No. 3 (NOAA-N22-126-72G), Dept. of Elec. Eng., Univ. of Colorado, Boulder (1973).

3. V.A. Kaplun and A.A. Pistol'kors, Radio Eng. Electron. Phys. v.9, pp. 1618-1626 (1964).

4. J.R. Wait, Radio Science, v. 7, pp. 675-679 (1972).

5. D.C. Chang and R.G. Olsen, Radio Science v. 10, pp. 823-831 (1975).

6. D.C. Chang and J.R. Wait, IEEE Trans. Commun. v. COM-22, pp. 421-427 (1974).

7. L.J. van der Pauw, Philips Res. Repts. v. 31, pp. 35-70 (1976).

8. G. Kowalski and R. Pregla, AEÜ, v. 27, pp. 125-130 (1973).

9. A.T. Fialkovskiy, Radio Eng. Electron. Phys. v. 21, no. 3, pp. 7 - 15 (1976).

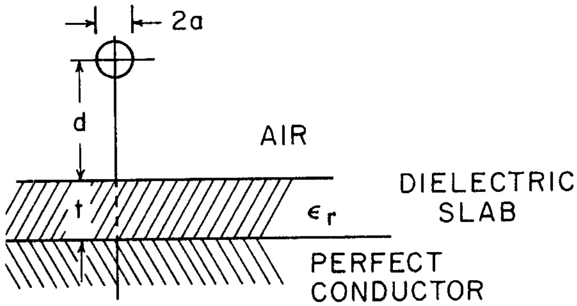

Fig. 1 Geometry of the problem.

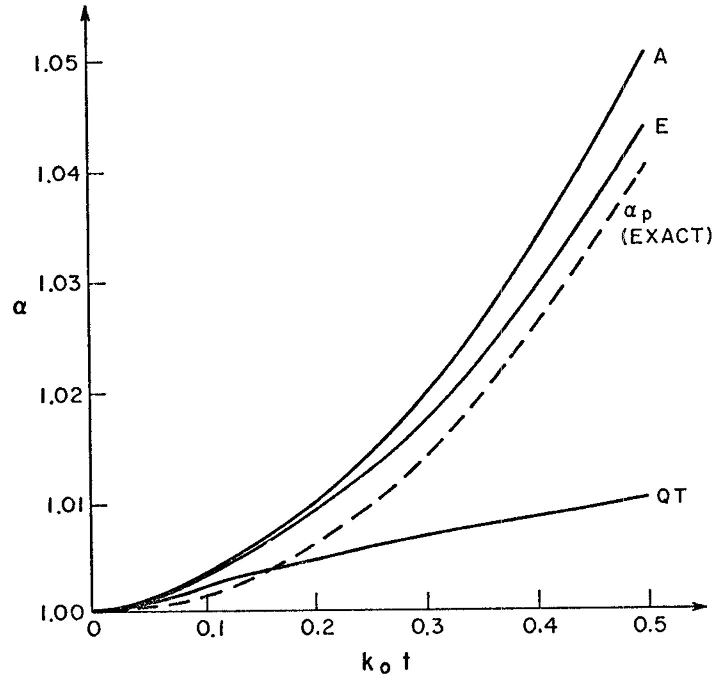

Fig. 2 Comparison of exact solution (E), solution of eqn.(10)(A), and quasi-TEM approximation (QT), for D = 1.0, n = 1.5.

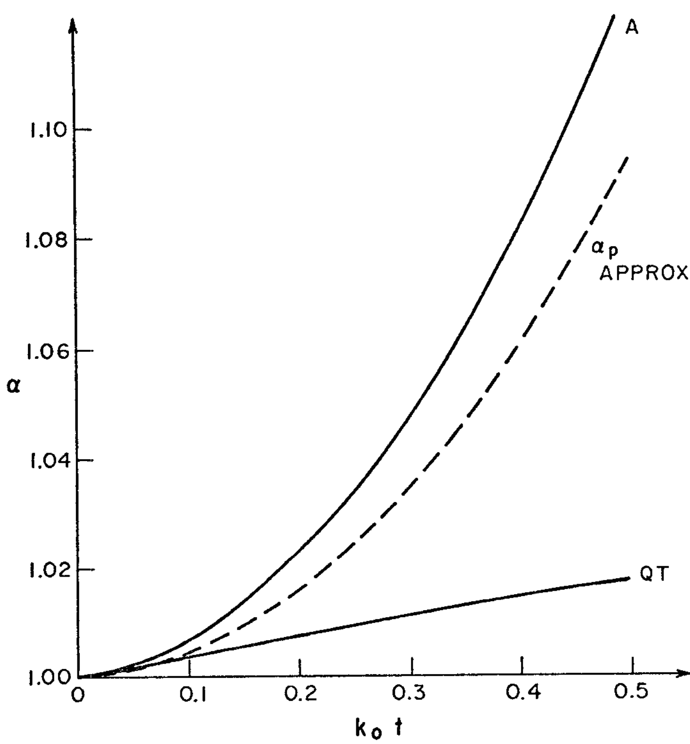

Fig. 3 Solution of eqn.(10)(A) and quasi-TEM approximation (QT) for D = 1.0, n = 3.0.

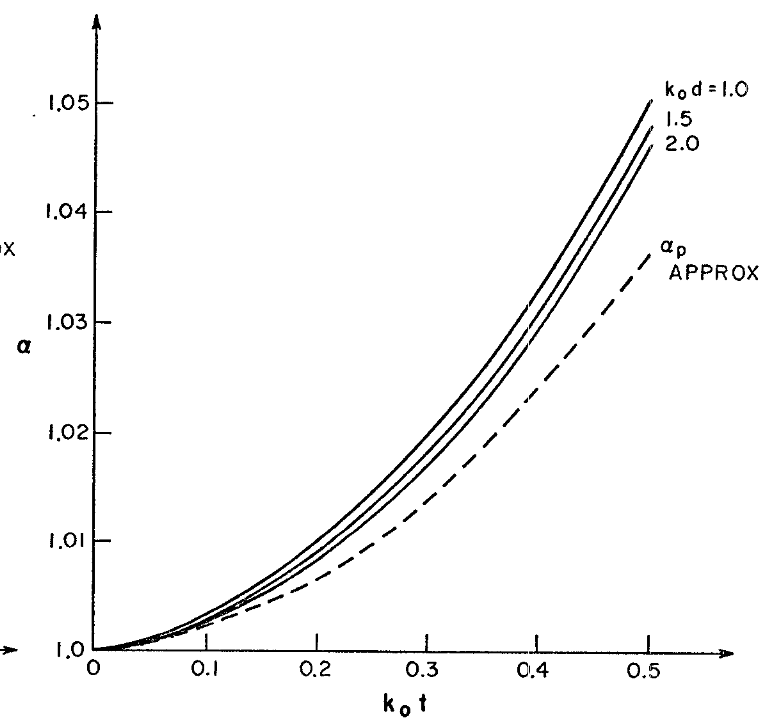

Fig. 4 Solution of eqn.(10) for various values of D, and u = 1.5.